

A numerical simulation method and analysis of a complete thermoacoustic-Stirling engine

Hong Ling^{a,b}, Ercang Luo^{a,*}, Wei Dai^a

^a Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, Beijing 100080, China

^b Graduate School of the Chinese Academy of Sciences, Beijing 100039, China

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Abstract

Thermoacoustic prime movers can generate pressure oscillation without any moving parts on self-excited thermoacoustic effect. The details of the numerical simulation methodology for thermoacoustic engines are presented in the paper. First, a four-port network method is used to build the transcendental equation of complex frequency as a criterion to judge if temperature distribution of the whole thermoacoustic system is correct for the case with given heating power. Then, the numerical simulation of a thermoacoustic-Stirling heat engine is carried out. It is proved that the numerical simulation code can run robustly and output what one is interested in. Finally, the calculated results are compared with the experiments of the thermoacoustic-Stirling heat engine (TASHE). It shows that the numerical simulation can agree with the experimental results with acceptable accuracy.

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1. Introduction

Actually, thermoacoustic phenomenon is very popular in nature and usually occurs when heat exchange or combustion takes place in acoustical cavities. Some thermoacoustic effects have attracted attention for over two centuries, but a quantitatively accurate understanding was not achieved until Rott's theoretical studies [1]. Later on, his theory has been confirmed experimentally by many researchers and has been riched very much. The thermoacoustic network model developed based on the Rott's linear thermoacoustic theory is a powerful tool to deal with all components of the thermoacoustic engine (TAE) [2] in a simpler way, with which it is possible to match the solutions of adjacent segments by using continuity of physical variables, such as oscillating pressure $p_1(x)$, oscillating volumetric velocity $U_1(x)$ and mean temperature $T_0(x)$. The computer code "Thermoacoustic simulator" [3] can be

used to analyze and design electrically-driven regenerative cryocoolers whose operating frequency is given in advance. For thermoacoustic heat engine, its resonant frequency is, however, unknown, the simulator does not work at all. The DeltaE code has been successfully developed to predict performance of various thermoacoustic systems by simultaneously integrating Rott's wave and energy-flow equations [4,5], but its numerical method for resolving questions needs an art-of-state simulation technique.

2. Formulation of numerical method

By expanding all thermodynamic and acoustic variables to the first order, i.e., $\zeta = \zeta_0 + \text{Re}[\zeta_1 e^{j\omega t}]$, we have the following expressions of the original thermoacoustic equations as[6]:

Wave equations:

$$\frac{dp_1}{dx} = -Z_F U_1 \quad (1a)$$

$$\frac{dU_1}{dx} = -Y_C p_1 + \frac{f_{WT}}{T_0} \frac{dT_0}{dx} U_1 \quad (1b)$$

* Corresponding author. Postal address: P.O. Box 2711, Beijing 100080, China. Fax.: +86 10 82543750.

E-mail address: ecluo@cl.cryo.ac.cn (E. Luo).

Energy equations:

$$\frac{dT_0}{dx} = \frac{\text{Re}[U_1 p_1^* (1 - f_{qx})]/2 - E_x}{A_f K_e} \quad (2a)$$

$$\frac{dE_x}{dx} = h_w \prod_w (T_e - T_0) \quad (2b)$$

Details of the deduction and the meanings of all parameters could be found in Ref. [6].

Making discretion for Eqs. (2a) and (2b), we can further have the following equations for describing the output parameters dependent of the inlet parameter for each thermodynamic element:

$$\begin{bmatrix} p_1(x_n) \\ U_1(x_n) \end{bmatrix} = \mathbf{F} \begin{bmatrix} p_1(x_0) \\ U_1(x_0) \end{bmatrix} \quad (3a)$$

$$\begin{bmatrix} T_0(x_n) \\ E_x(x_n) \end{bmatrix} = \mathbf{G} \begin{bmatrix} T_0(x_0) \\ E_x(x_0) \end{bmatrix} + \mathbf{S} \quad (3b)$$

where $\mathbf{F}$ is a 2×2 flowing transmission matrix (FTM), $\mathbf{G}$ is a 2×2 thermal transmission matrix (TTM) and $\mathbf{S}$ is a 2×1 thermal response function vector (TRFV).

According to Eqs. (3a) and (3b), the flowing variables p_1 , U_1 and thermodynamic variables T_0 , E_x could be regarded as two four-port networks with two inputs and outputs, respectively. Theoretically, once the FTM, TTM and TRFV of all components are obtained, the whole system is determined. However, there are complicated coupling interactions between the wave and thermodynamic networks, and these acoustical and thermodynamic parameters cannot be obtained immediately without iteration.

Before starting the simulation, the resonant frequency and the temperature distribution of a thermoacoustic engine must be predicted or given as an initial value. For a known temperature distribution and acoustical boundary condition, we always can build a transcendental equation of resonant frequency as follows:

$$F(T_0, p_1, U_1; \omega, \text{gas properties, geometry}) = 0. \quad (4)$$

By solving it we can obtain the frequency ω , whose imaginary part must be zero. Otherwise it is thought that a correct distribution of temperature is not achieved and a new iteration has to be made again.

3. Simulation and analysis of a thermoacoustic-Stirling engine

A calculation sample is given for a thermoacoustic-Stirling heat engine built in our laboratory, which is schematically shown in Fig. 1. The dimensions of the main parts are

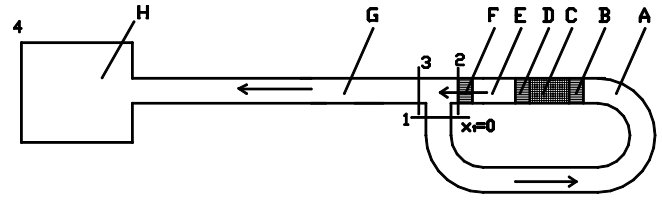


Fig. 1. Schematic diagram of the thermoacoustic-Stirling engine. (A: Feedback inertia tube B: Main cold heat exchanger (CHX) C: Regenerator D: Hot heat exchanger (HHX) E: Thermal buffer tube (TBT) F: Secondary cold heat exchanger G: Resonator tube H: Reservoir).

tabulated in Table 1. The three heat exchangers have the same structure with many parallel channels formed by copper fins. The fin gaps are 1.0 mm and 0.5 mm for the heater and the cooler, respectively, in which their temperature is anchored at 288 K by circulating water. The regenerator is made from stainless steel screens of 60 mesh. The feedback tube includes two segments and a transition cone of 5 cm length, which is not shown in Fig. 1. The working gas is pure helium with a purity of 99.99%.

Some interesting results are presented in Figs. 2–4, and the sub-diagram is the partial zoom of dashed frame. The operating conditions are: a mean pressure of 21.5 bar and a heat power of 1188 W. Fig. 2a shows that the pressure amplitude in the looped tube keeps on a high level and changes a little. The maximum value appears near the cold end of the regenerator in which there is an obvious decrease resulting from its large resistance. In the resonator, the pressure amplitude decreases to zero (pressure node) and then rises reversely in the reservoir. Fig. 2b shows that along the feedback tube the volume velocity decreases from the maximum (section 1 in Fig. 1) to almost zero at the cold end of the regenerator, then it is amplified in the regenerator with counter phase (as seen clearly in sub-diagram of Fig. 2b). The regenerator works as a volumetric velocity source, where the velocity is just as small as to enlarge the impedance (p/U_1) and to reduce viscous flowing loss. It is the reason why a thermoacoustic-Stirling heat engine can be more efficient than a pure traveling-wave engine [7].

Fig. 3 shows that an acoustic power flux must be feed-backed into the ambient end of the regenerator and then be amplified. It divaricates in the T-joint where one branch goes into the feedback tube and the other enters the resonator. In the abrupt section between the resonator tube and the reservoir, nonlinear loss happens and causes a sudden decrease of work flux and a temperature rise. Fig. 3 also shows that total energy keeps constant in both the regenerator

Table 1
Dimensions of the simulation engine (units: mm)

Items	Feedback tube (2)		After-cooler (2)		Regenerator	Heater	TBT	Resonator tube	Reservoir
Diameter	100	68	80	80	80	80	80	50	150
Length	536	440	45	22	80	58	220	3188	515

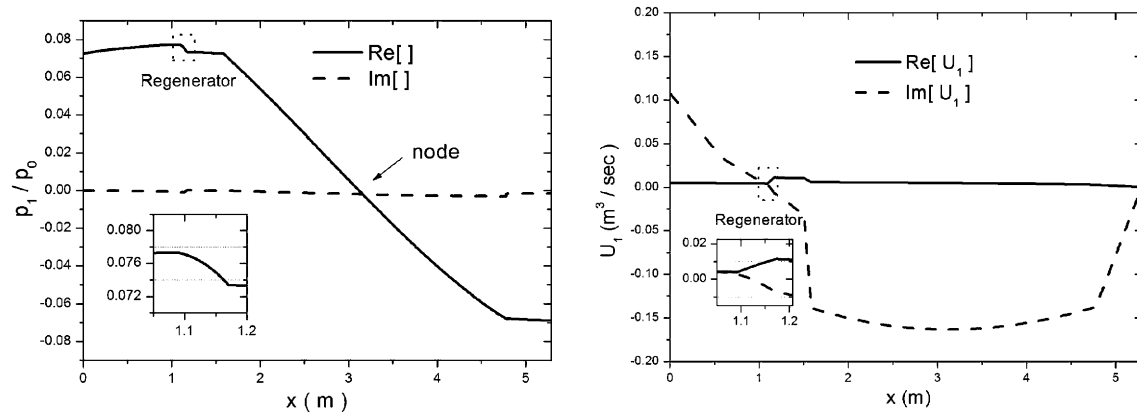


Fig. 2. The calculated axial distribution of relative pressure amplitude and volumetric velocity. (a) Pressure; (b) Velocity.

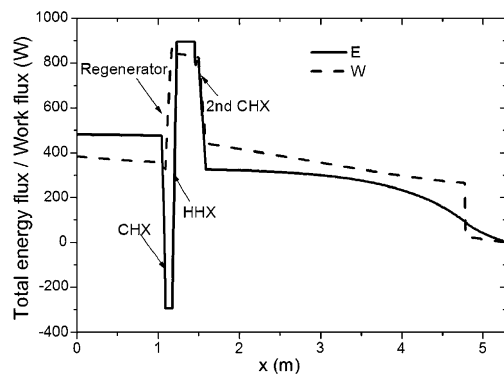


Fig. 3. The calculated axial distribution of total energy flux and work flux.

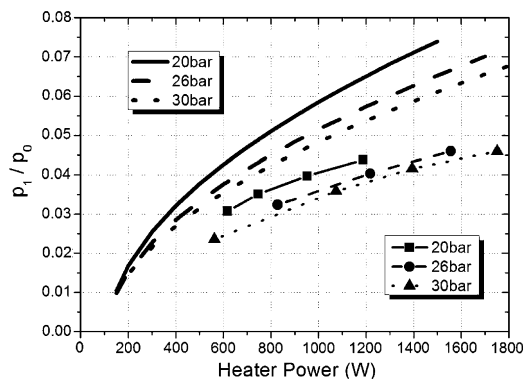


Fig. 4. Relative pressure amplitude versus heater power under three charging pressures.

ator and TBT owning that there is no energy exchange with surroundings.

Fig. 4 presents a comparison of the pressure amplitudes between the calculations and experiments under three charging pressures of 20 bar, 26 bar, and 30 bar. The monitored point is located at the middle of the feedback tube (about 0.536 m apart from the axial origin), where a pressure sensor is mounted. The calculation curves basically reflect experimental phenomena and indicate that the oscil-

lation will be strengthened with heater power increase. It is found that the system with higher charging pressure can produce as much powerful pressure ratio as lower pressure only with more heating power. The solid symbols shown in Fig. 4 are experimental points that give about 70% of the calculation results. The amplitude is influenced by the all sorts of loss, which cannot be solved by linear theory only with some simple nonlinear corrections. Regarding the resonant frequency, the numerical simulation is in good agreement with experiment. The difference between them is no more than 0.5 Hz, and the relative error is within 1%.

4. Conclusions

A numerical simulation methodology for a complete thermoacoustic engine is described in this paper. In the numerical simulation, the complex resonant frequency is a key parameter that is used to judge if a self-excited thermoacoustic engine reaches stable statues. In addition, the wave and energy equations are decoupled to solve. Based on the methodology, a simulation code, which is able to simulate various self-excited thermoacoustic engines, is developed. Particularly, the numerical simulation of a thermoacoustic-Stirling heat engine is made as an example. Then, the operating mechanism of the thermoacoustic-Stirling heat engine is analyzed with the numerical simulation code. The simulation result shows that the thermoacoustic-Stirling heat engine previously developed in our laboratory can operate mainly on traveling-wave mode and undergo a thermodynamic cycle similar to the Stirling cycle. Finally, comparison between the numerical simulation and previous experiment has been made, demonstrating that the code runs robustly and gives an acceptable accuracy.

Acknowledgments

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